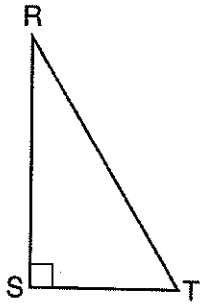
Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. [48]

Use this space for computations.

1 Which object is formed when right triangle RST shown below is rotated around leg $\overline{RS}$?



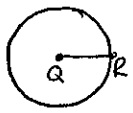
(1) a pyramid with a square base
(2) an isosceles triangle
(3) a right triangle
(4) a cone

2 The vertices of $\triangle JKL$ have coordinates $J(5,1)$, $K(-2,-3)$, and $L(-4,1)$. Under which transformation is the image $\triangle J'K'L'$ not congruent to $\triangle JKL$?

(1) a translation of two units to the right and two units down
(2) a counterclockwise rotation of 180 degrees around the origin
(3) a reflection over the x-axis
(4) a dilation with a scale factor of 2 and centered at the origin

3 The center of circle Q has coordinates $(3,-2)$. If circle Q passes through $R(7,1)$, what is the length of its diameter?

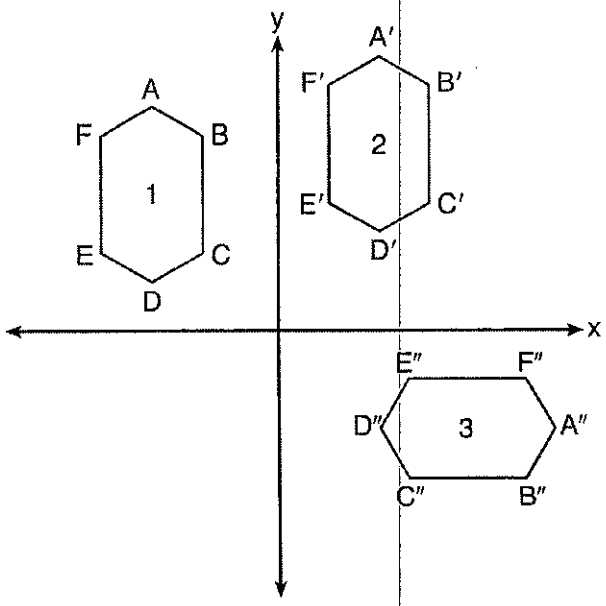
(1) 50
(2) 25
(3) 10
(4) 5



$2\sqrt{(7-3)^2+(1-2)^2}$
$2\sqrt{4^2+3^2}$
$2\sqrt{16+9}$
$2\sqrt{25}$
$2(5)$
10

Use this space for computations.

4 In the diagram below, congruent figures 1, 2, and 3 are drawn.



Which sequence of transformations maps figure 1 onto figure 2 and then figure 2 onto figure 3?

(1) a reflection followed by a translation

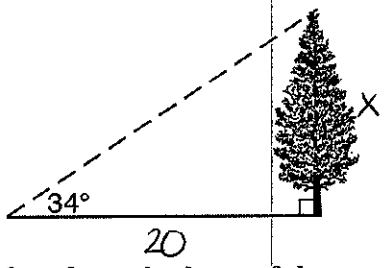
(2) a rotation followed by a translation

(3) a translation followed by a reflection

● a translation followed by a rotation

5 As shown in the diagram below, the angle of elevation from a point on the ground to the top of the tree is 34°.

$\frac{\tan 34}{1} = \frac{X}{20}$



If the point is 20 feet from the base of the tree, what is the height of the tree, to the *nearest tenth of a foot*?

(1) 29.7

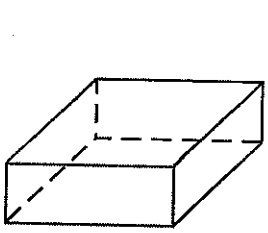
(2) 16.6

● 13.5

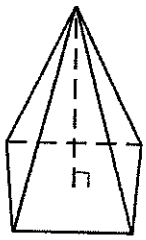
(4) 11.2

6 Which figure can have the same cross section as a sphere?

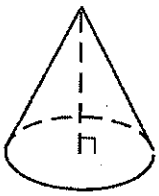
Use this space for computations.



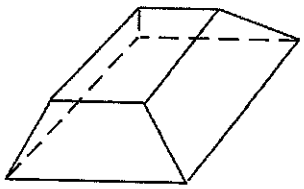
(1)



(3)



●



(4)

7 A shipping container is in the shape of a right rectangular prism with a length of 12 feet, a width of 8.5 feet, and a height of 4 feet. The container is completely filled with contents that weigh, on average, 0.25 pound per cubic foot. What is the weight, in pounds, of the contents in the container?

(1) 1,632

(2) 408

● 102

(4) 92

$V = lwh$

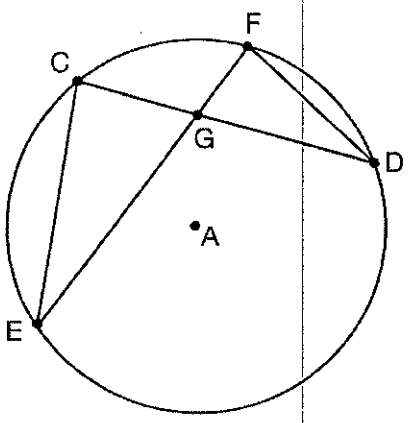
$V = (12)(8.5)(4)$

$V = 408$

$408(.25) = 102$

Use this space for computations.

8 In the diagram of circle A shown below, chords $\overline{CD}$ and $\overline{EF}$ intersect at G, and chords $\overline{CE}$ and $\overline{FD}$ are drawn.



Which statement is *not* always true?

● $\overline{CG} \cong \overline{FG}$

(2) $\angle CEG \cong \angle FDG$

(3) $\frac{CE}{EG} = \frac{FD}{DG}$

(4) $\triangle CEG \sim \triangle FDG$

9 Which equation represents a line that is perpendicular to the line represented by $2x - y = 7$?

● $y = -\frac{1}{2}x + 6$

(2) $y = \frac{1}{2}x + 6$

(3) $y = -2x + 6$

(4) $y = 2x + 6$

$2x - y = 7$
$-2x \quad -2x$
$-y = -2x + 7$
$\frac{-y}{-1} = \frac{-2x+7}{-1}$
$y = 2x - 7$

$2 \quad 1 \quad -\frac{1}{2}$

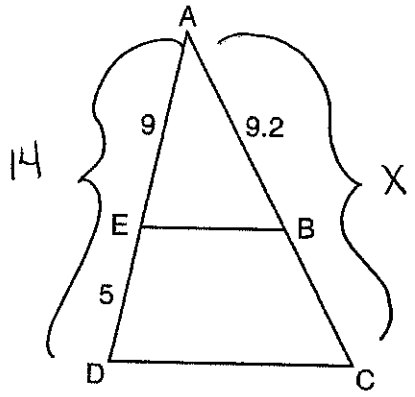
10 Which regular polygon has a minimum rotation of 45° to carry the polygon onto itself?

Use this space for computations.

● octagon
(2) decagon
(3) hexagon
(4) pentagon

$\frac{360}{45} = 8$

11 In the diagram of $\triangle ADC$ below, $\overline{EB} \parallel \overline{DC}$, $AE = 9$, $ED = 5$, and $AB = 9.2$.



$\frac{9}{14} = \frac{9.2}{X}$

$\frac{9X}{9} = \frac{128.8}{9}$

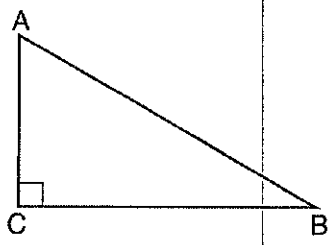
$X = 14.3$

What is the length of $\overline{AC}$, to the *nearest tenth*?

(1) 5.1
● 14.3
(2) 5.2
(4) 14.4

Use this space for computations.

12 In scalene triangle ABC shown in the diagram below, $m\angle C = 90°$.



Which equation is always true?

(1) $\sin A = \sin B$

(2) $\cos A = \cos B$

(3) $\cos A = \sin C$

● $\sin A = \cos B$

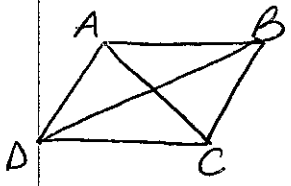
13 Quadrilateral $ABCD$ has diagonals $\overline{AC}$ and $\overline{BD}$. Which information is *not* sufficient to prove $ABCD$ is a parallelogram?

(1) $\overline{AC}$ and $\overline{BD}$ bisect each other.

(2) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$

(3) $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$

● $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$



14 The equation of a circle is $x^2 + y^2 + 6y = 7$. What are the coordinates of the center and the length of the radius of the circle?

(1) center (0,3) and radius 4

● center (0,−3) and radius 4

(3) center (0,3) and radius 16

(4) center (0,−3) and radius 16

$x^2 + y^2 + 6y + 9 = 7 + 9$

$x^2 + (y+3)^2 = 16$

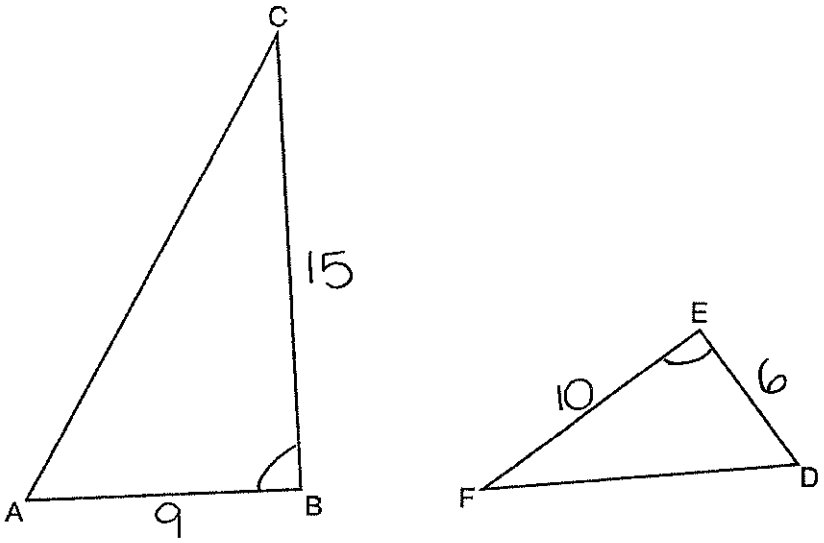
$C = (0, -3)$

$r = 4$

$\left(\frac{6}{2}\right)^2 = (3)^2 = 9$

Use this space for computations.

15 Triangles ABC and DEF are drawn below.



If $AB = 9$, $BC = 15$, $DE = 6$, $EF = 10$, and $\angle B \cong \angle E$, which statement is true?

(1) $\angle CAB \cong \angle DEF$

(2) $\frac{AB}{CB} = \frac{FE}{DE}$

● $\triangle ABC \sim \triangle DEF$

(4) $\frac{AB}{DE} = \frac{FE}{CB}$

16 If $\triangle ABC$ is dilated by a scale factor of 3, which statement is true of the image $\triangle A'B'C'$?

(1) $3A'B' = AB$

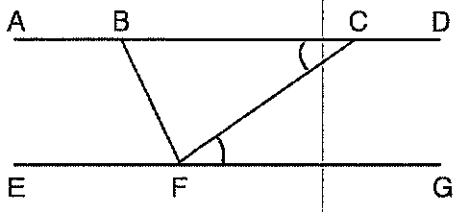
● $B'C' = 3BC$

(3) $m\angle A' = 3(m\angle A)$

(4) $3(m\angle C') = m\angle C$

Use this space for computations.

17 Steve drew line segments $ABCD$, EFG, BF, and CF as shown in the diagram below. Scalene $\triangle BFC$ is formed.



Which statement will allow Steve to prove $\overline{ABCD} \parallel \overline{EFG}$?

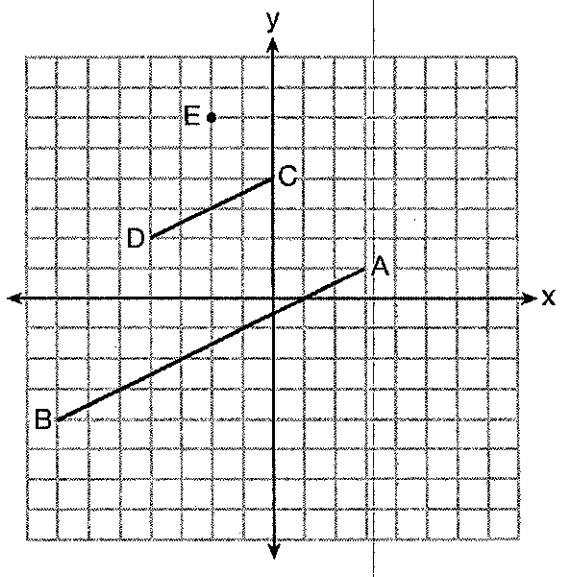
● $\angle CFG \cong \angle FCB$

(3) $\angle EFB \cong \angle CFB$

(2) $\angle ABF \cong \angle BFC$

(4) $\angle CBF \cong \angle GFC$

18 In the diagram below, $\overline{CD}$ is the image of $\overline{AB}$ after a dilation of scale factor k with center E.



Which ratio is equal to the scale factor k of the dilation?

● $\frac{EC}{EA}$

(3) $\frac{EA}{BA}$

(2) $\frac{BA}{EA}$

(4) $\frac{EA}{EC}$

Use this space for computations.

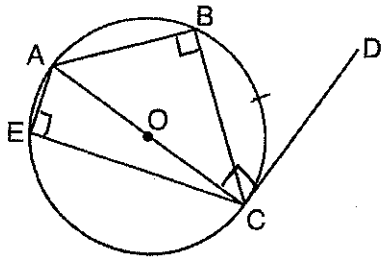
19 A gallon of paint will cover approximately 450 square feet. An artist wants to paint all the outside surfaces of a cube measuring 12 feet on each edge. What is the *least* number of gallons of paint he must buy to paint the cube?

(1) 1
(2) 2
(3) 3
(4) 4

$SA = 6(12)(12)$

$\frac{864}{450} = 1.92$

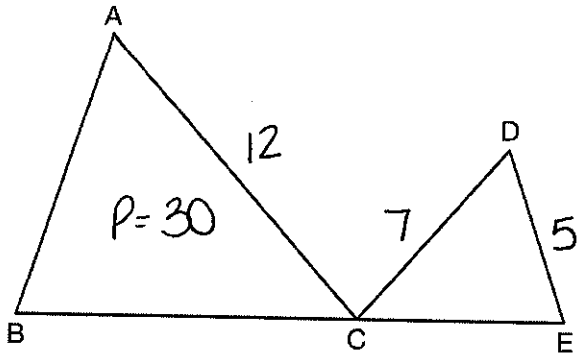
20 In circle O shown below, diameter $\overline{AC}$ is perpendicular to $\overline{CD}$ at point C, and chords $\overline{AB}$, $\overline{BC}$, $\overline{AE}$, and $\overline{CE}$ are drawn.



Which statement is *not* always true?

(1) $\angle ACB \cong \angle BCD$
(2) $\angle ABC \cong \angle ACD$
(3) $\angle BAC \cong \angle DCB$
(4) $\angle CBA \cong \angle AEC$

21 In the diagram below, $\triangle ABC \sim \triangle DEC$.



$\frac{12}{30} = \frac{7}{x}$

$12x = 210$

$x = 17.5$

If $AC = 12$, $DC = 7$, $DE = 5$, and the perimeter of $\triangle ABC$ is 30, what is the perimeter of $\triangle DEC$?

(1) 12.5
(2) 14.0
(3) 14.8
(4) 17.5

$y = \frac{-2}{3}x + \frac{8}{3}$

Use this space for computations.

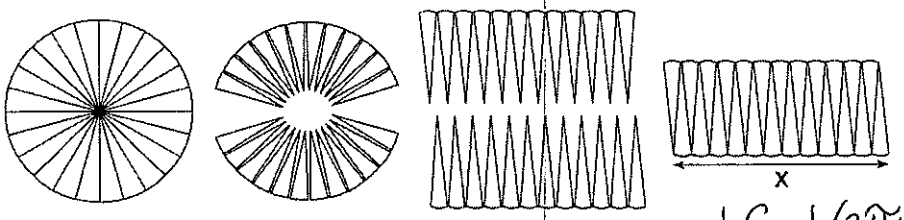
22 The line $3y = -2x + 8$ is transformed by a dilation centered at the origin. Which linear equation could be its image?

(1) $2x + 3y = 5$ (3) $3x + 2y = 5$

(2) $2x - 3y = 5$ (4) $3x - 2y = 5$

$3y + 2x = 8$ same slope

23 A circle with a radius of 5 was divided into 24 congruent sectors. The sectors were then rearranged, as shown in the diagram below.



$\frac{1}{2}C = \frac{1}{2}(2\pi r)$

$\frac{1}{2}(2)(\pi)(5)$

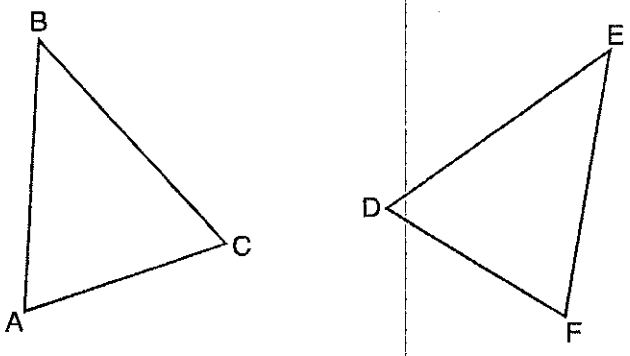
5π

To the *nearest integer*, the value of x is

(1) 31 (3) 12

(2) 16 (4) 10

24 Which statement is sufficient evidence that $\triangle DEF$ is congruent to $\triangle ABC$?



(1) $AB = DE$ and $BC = EF$

(2) $\angle D \cong \angle A$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

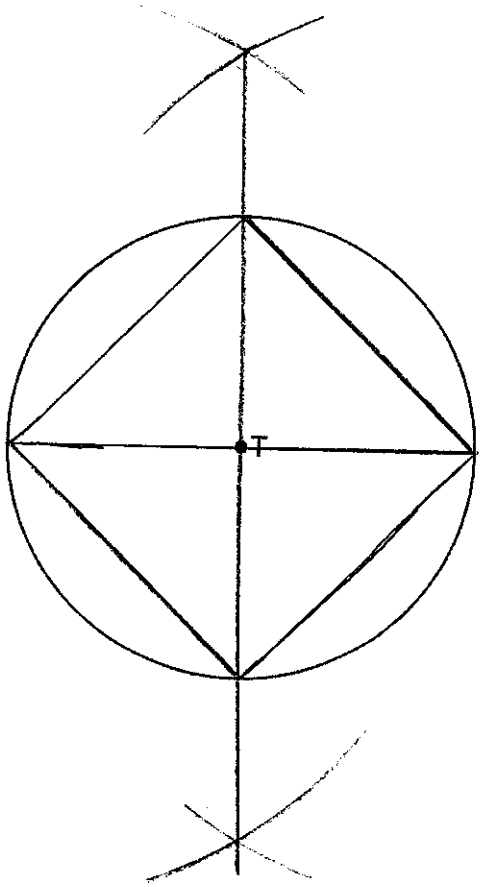
(3) There is a sequence of rigid motions that maps $\overline{AB}$ onto $\overline{DE}$, $\overline{BC}$ onto $\overline{EF}$, and $\overline{AC}$ onto $\overline{DF}$.

(4) There is a sequence of rigid motions that maps point A onto point D, $\overline{AB}$ onto $\overline{DE}$, and $\angle B$ onto $\angle E$.

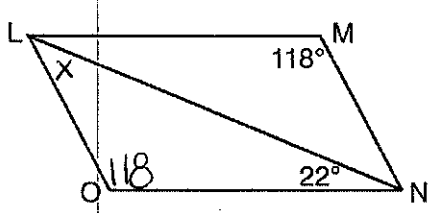
Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

25 Use a compass and straightedge to construct an inscribed square in circle T shown below. [Leave all construction marks.]



26 The diagram below shows parallelogram $LMNO$ with diagonal $\overline{LN}$, $m\angle M = 118°$, and $m\angle LNO = 22°$.



Explain why $m\angle NLO$ is 40 degrees.

$\angle M \cong \angle O$ in a llogram opposite $\angle$s are $\cong$

angles of a $\triangle$ add up to 180°

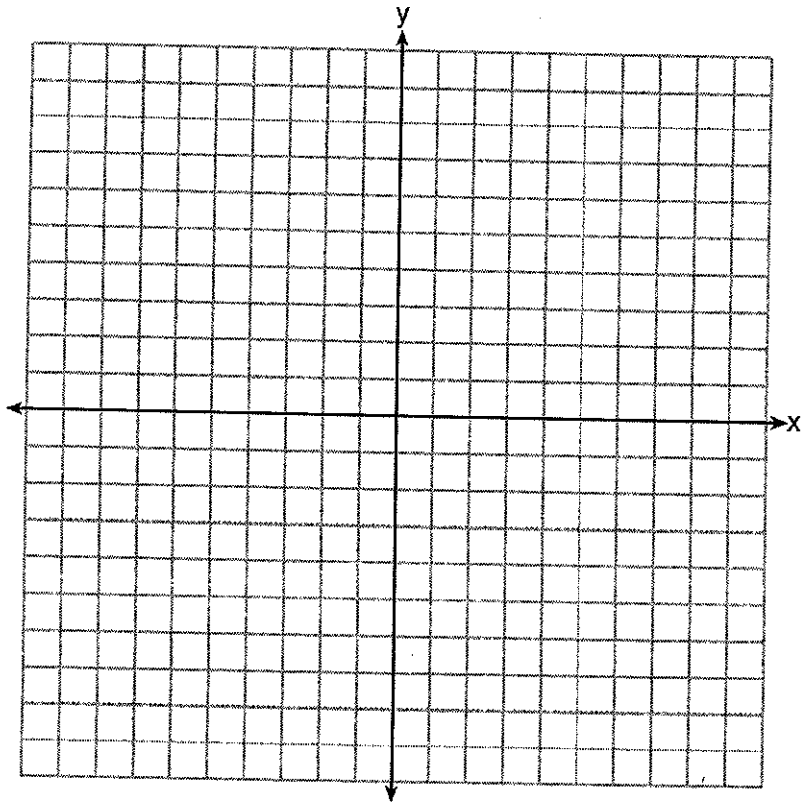
$X + 22 + 118 = 140 + X = 180$

$X = 40$

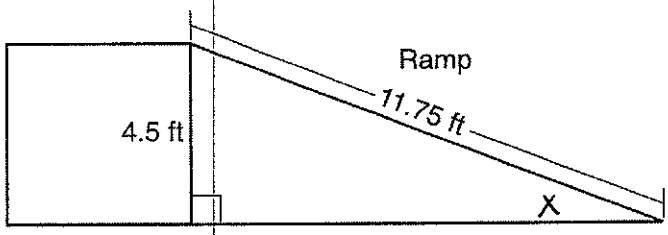
27 The coordinates of the endpoints of $\overline{AB}$ are $A(-6,-5)$ and $B(4,0)$. Point P is on $\overline{AB}$. Determine and state the coordinates of point P, such that $AP:PB$ is 2:3. $\frac{2}{5}$
[The use of the set of axes below is optional.]

$A(-6,-5) \xrightarrow{T_{10,5}} B(4,0)$

$A(-6,-5) \xrightarrow[T_{4,2}]{T_{10(\frac{2}{5}),5(\frac{2}{5})}} \boxed{P(-2,-3)}$



28 The diagram below shows a ramp connecting the ground to a loading platform 4.5 feet above the ground. The ramp measures 11.75 feet from the ground to the top of the loading platform.

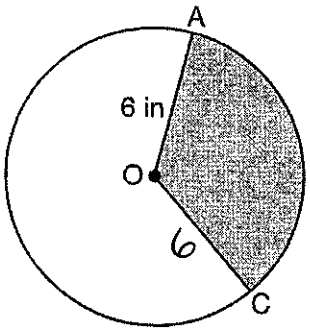


Determine and state, to the *nearest degree*, the angle of elevation formed by the ramp and the ground.

$\sin X = \frac{4.5}{11.75}$

$X = 23^\circ$

29 In the diagram below of circle O, the area of the shaded sector AOC is 12π in^2 and the length of $\overline{OA}$ is 6 inches. Determine and state m$\angle AOC$.



$$12\pi = \frac{n}{360} \cdot \pi r^2$$

$$12\pi = \frac{n}{360} \cdot \pi (6)^2$$

$$\frac{12\pi}{1} = \frac{36\pi n}{360}$$

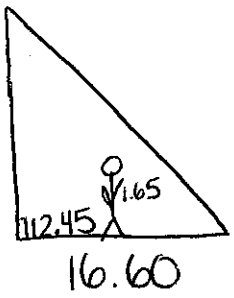
$$\frac{4320\pi}{36\pi} = \frac{36\pi n}{36\pi}$$

$$\boxed{120^\circ = n}$$

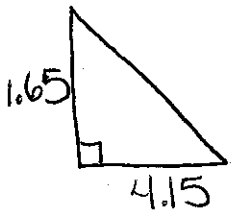
30 After a reflection over a line, $\triangle A'B'C'$ is the image of $\triangle ABC$. Explain why triangle ABC is congruent to triangle $A'B'C'$.

a line reflection is a rigid motion so it preserves distance and angle measure

31 A flagpole casts a shadow 16.60 meters long. Tim stands at a distance of 12.45 meters from the base of the flagpole, such that the end of Tim's shadow meets the end of the flagpole's shadow. If Tim is 1.65 meters tall, determine and state the height of the flagpole to the *nearest tenth of a meter.*

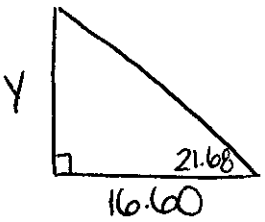


$$16.60 - 12.45 = 4.15$$



$$\tan X = \frac{1.65}{4.15}$$

$$X = 21.6823$$



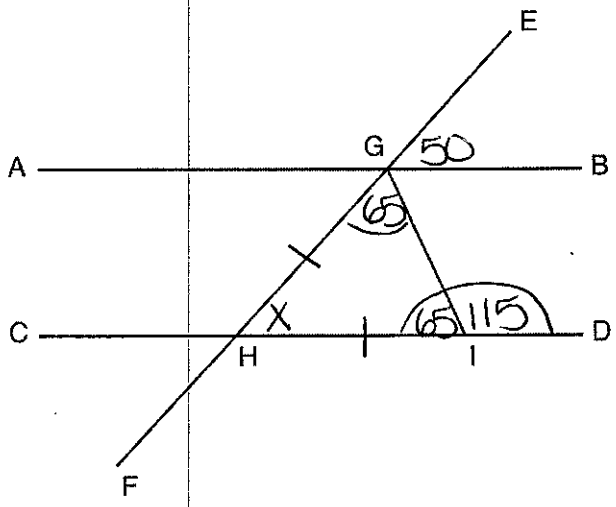
$$\tan 21.68 = \frac{Y}{16.60}$$

$$\boxed{Y = 6.6}$$

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

32 In the diagram below, $\overline{EF}$ intersects $\overline{AB}$ and $\overline{CD}$ at G and H, respectively, and $\overline{GI}$ is drawn such that $\overline{GH} \cong \overline{IH}$.



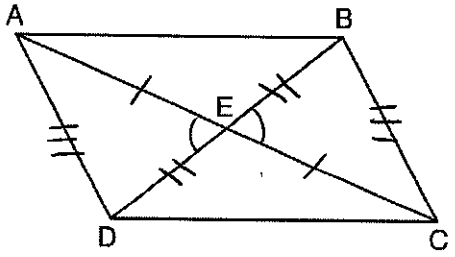
180
-115
65

If $m\angle EGB = 50°$ and $m\angle DIG = 115°$, explain why $\overline{AB} \parallel \overline{CD}$.

X+65+65=180

X+

33 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at E

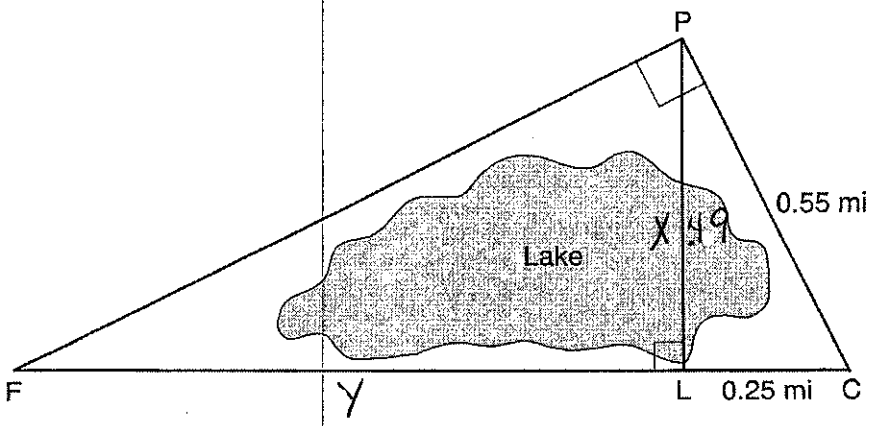


Prove: $\triangle AED \cong \triangle CEB$

Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

R_{180} around point E that maps A onto C
E onto E
D onto B

34 In the diagram below, the line of sight from the park ranger station, P, to the lifeguard chair, L, on the beach of a lake is perpendicular to the path joining the campground, C, and the first aid station, F. The campground is 0.25 mile from the lifeguard chair. The straight paths from both the campground and first aid station to the park ranger station are perpendicular.



If the path from the park ranger station to the campground is 0.55 mile, determine and state, to the *nearest hundredth of a mile*, the distance between the park ranger station and the lifeguard chair.

$X^2 + .25^2 = .55^2$

$X^2 + .0625 = .3025$

$X^2 = .24$

$X = .49$

Gerald believes the distance from the first aid station to the campground is at least 1.5 miles. Is Gerald correct? Justify your answer.

$\frac{Y}{.49} = \frac{.49}{.25}$

$.25Y = .2401$

$Y = .9604$

$.9604 + .25 = 1.2104$

No

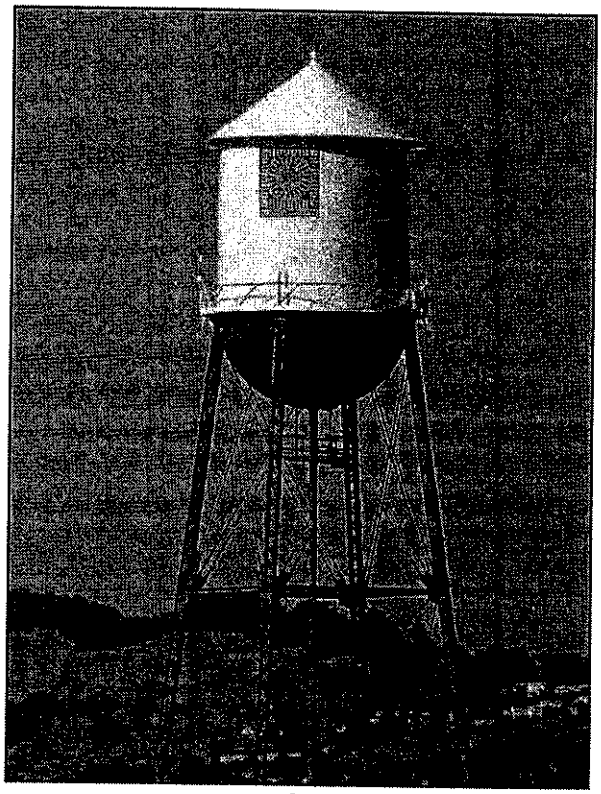
Part IV

Answer the 2 questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

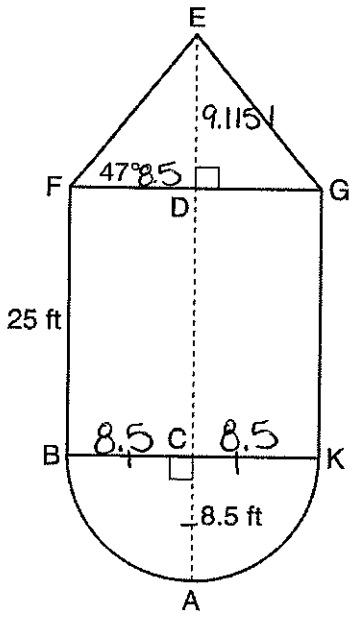
35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.

$\tan 47 = \frac{X}{8.5}$

$X = 9.1151$



Source: http://en.wikipedia.org



Question 35 is continued on the next page.

Question 35 continued

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47°$, determine and state, to the *nearest cubic foot*, the volume of the water tower.

$V_{hemisphere} = \frac{1}{2} \cdot \frac{4}{3}\pi r^3$

$\frac{4}{6}\pi(8.5)^3 = 1286.2204$

409.4167π

$V_{cylinder} = \pi r^2 h$

$\pi(8.5)^2(25) = 5674.5017$

1806.25π

$V_{cone} = \frac{1}{3}\pi r^2 h$

$\frac{1}{3}\pi(8.5)^2(9.1151) = 689.6487$

219.52199π

7650 ft^3

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

$7650(62.4) = 477360\ (.85) = 405756$ NO

36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$. Prove that $\triangle RST$ is a right triangle.
[The use of the set of axes on the next page is optional.]

$RS = \frac{-1-4}{6-1} = \frac{3}{5}$

$ST = \frac{-4-6}{1-5} = \frac{-10}{6} = -\frac{5}{3}$

$\frac{3}{5} \cdot -\frac{5}{3} = -1$

$\perp$ lines form a right angle

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

$P(0,9)$

Question 36 is continued on the next page.

Question 36 continued

Prove that your quadrilateral *RSTP* is a rectangle.
[The use of the set of axes below is optional.]

$RS = \frac{3}{5}$

$ST = \frac{-5}{3}$

$TP = \frac{9-6}{0--5} = \frac{3}{5}$

$PR = \frac{9--1}{0-6} = \frac{10}{-6} = \frac{5}{-3}$

opp sides ||
consecutive sides ⊥